A local synaptic balancing rule for homeostatic plasticity

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Summary

Here we explore a local, balancing synaptic update rule for homeostatic plasticity in recurrent networks.

- We propose a mathematical framework for synaptic dynamics that minimizes a general cost function over the synaptic weights while maintaining network functionality.
- This model has attractive mathematical properties such as locality, isospectrality, and convexity.
- This rule predicts that neurons locally balance the cost of input and output and output synapses.

Homeostatic Plasticity

Homeostatic plasticity is a form of synaptic plasticity that is thought to stabilize a network from potential runaway positive feedback induced by Hebbian-type learning. [1]

Hypotheses for mechanisms of homeostatic plasticity include:

- ***heterosynaptic plasticity:*** changes at synapses not active during Hebbian plasticity [2]
- ***synaptic scaling:*** excitatory input synapses are scaled up or down by a common factor in response to a perturbation to recover a set-point firing rate. [3,4]

Dynamics

Observation: linear or ReLu network dynamics are simply rescaled under similarity transformation $J \to UJU^{-1}$

$$\tau\dot{\mathbf{x}} = -\mathbf{x} + \mathbf{J}\phi(\mathbf{x}) \longrightarrow \tau\dot{\tilde{\mathbf{x}}} = -\tilde{\mathbf{x}} + \mathbf{J}\phi(\tilde{\mathbf{x}}) \quad \text{with} \quad \tilde{\mathbf{x}} = \mathbf{U}^{-1}\mathbf{x}$$

Are some $\boldsymbol{J}$ matrices on this manifold more energetically favorable? Assume metabolic cost function of the form

$$C = \sum_{ij} |J_{ij}|^p$$

We propose a model of synaptic dynamics that moves along the manifold of isomorphic dynamics:

$$\mathbf{J}(t) = e^{-\mathbf{H}}\mathbf{J}^0 e^{\mathbf{H}} \quad \text{or} \quad J_{ij}(t) = J^0_{ij}\, e^{h_j(t)-h_i(t)}$$

where $h_i = H_{ii}$. Taking the time derivative, we find

$$\dot{J}_{ij} = J_{ij}(g_j - g_i) \quad \text{with} \quad g_i = \dot{h}_i$$

(This is a special case of a *Lax dynamical system*, $\dot{J} = [J, G]$, with diagonal G)

Choose the form of $\{h_i\}$ such that the time evolution in (2) minimizes the total cost C via gradient descent.

$$\dot{h}_i = -\frac{\partial C}{\partial h_i} \implies g_i = -\frac{\partial C}{\partial h_i}$$

we find:

$$g_k = p\left(\sum_j |J_{kj}|^p - \sum_i |J_{ik}|^p\right)$$

at equilibrium, $\dot{J} = 0 \implies g_k = 0 \ \forall\ k$

Each neuron balances the sum of its incoming and outgoing synaptic costs.

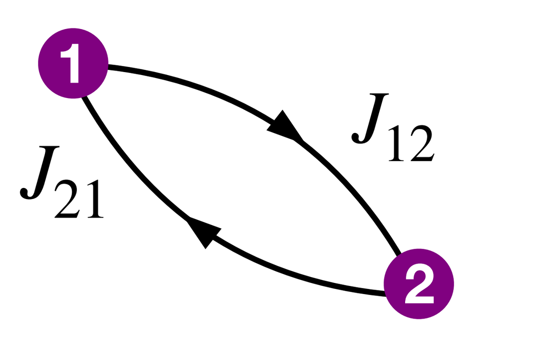
Properties

Convexity: can show that the cost function is convex in $\{h_i\}$ regardless of the value of p.

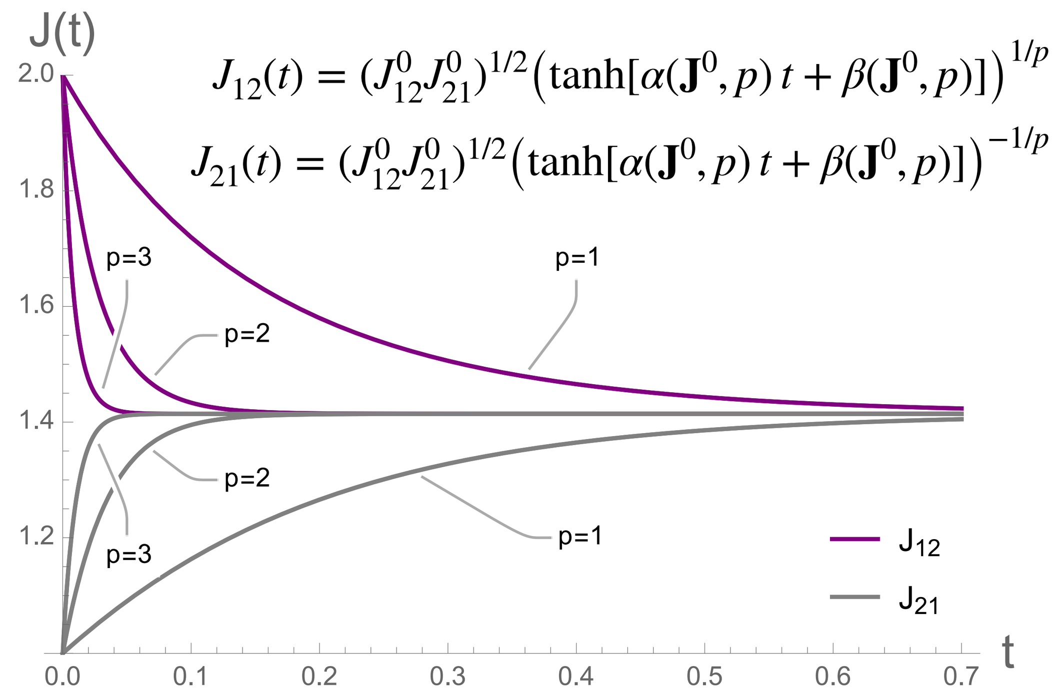
$$\frac{\partial^2 C}{\partial \mathbf{h}^2} = p^2\mathbf{L}$$

where $\mathbf{L}$ is the graph Laplacian of the symmetric matrix $\bar{c}_{ij} = c_{ij} + c_{ji}$ with $c_{ij} = |J_{ij}|^p$. Graph Laplacian $\implies$ positive semidefinite.

Two-neuron network: we can solve the dynamics exactly.

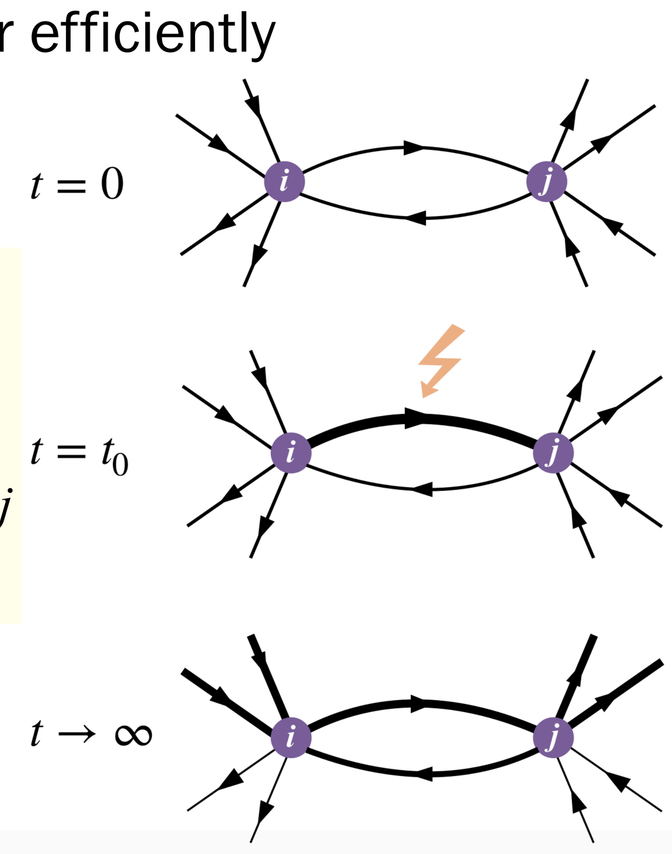


Two-neuron network becomes symmetric as $t \to \infty$.



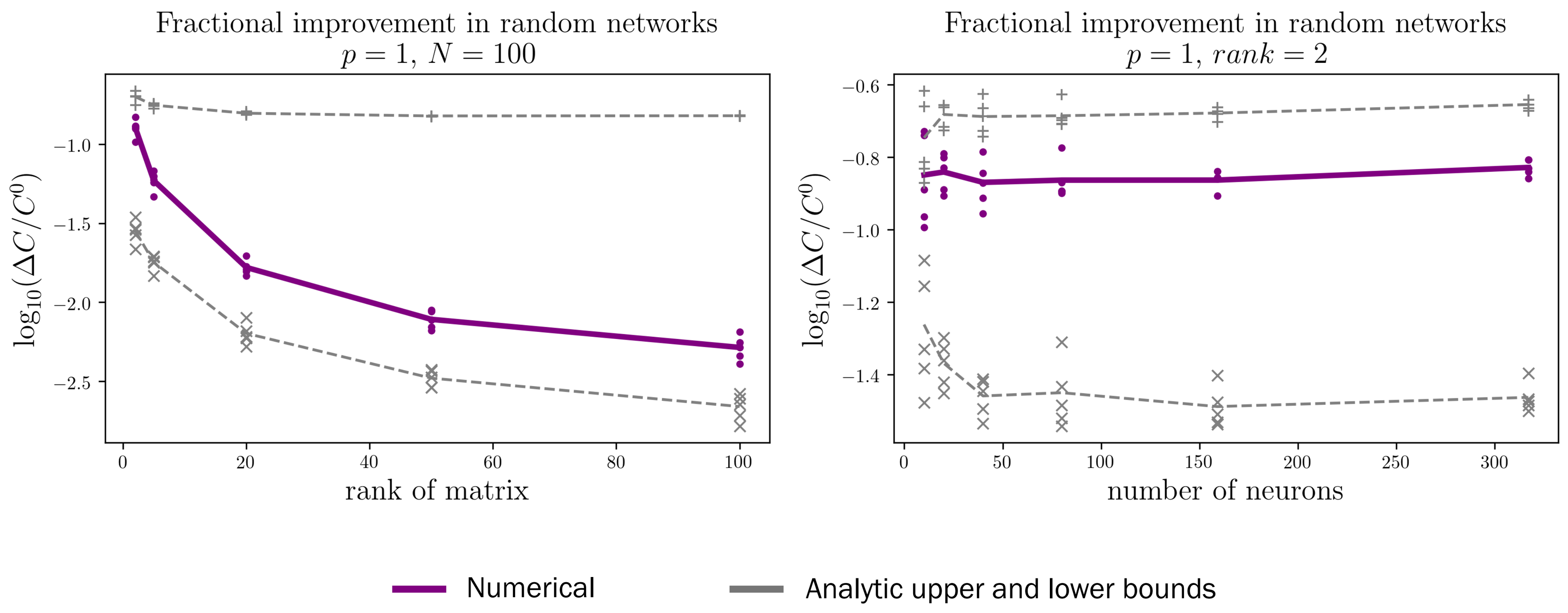
Response to perturbations: we can expand the dynamics about a fixed point to linear order and find rules for efficiently redistributing a perturbation

- Weaken J_{ij} and strengthen J_{ji}
- Strengthen the synapses incoming to neuron i
- Weaken the synapses outgoing from neuron i
- Strengthen the synapses outgoing from neuron j
- Weaken the synapses incoming to neuron j



Numerics

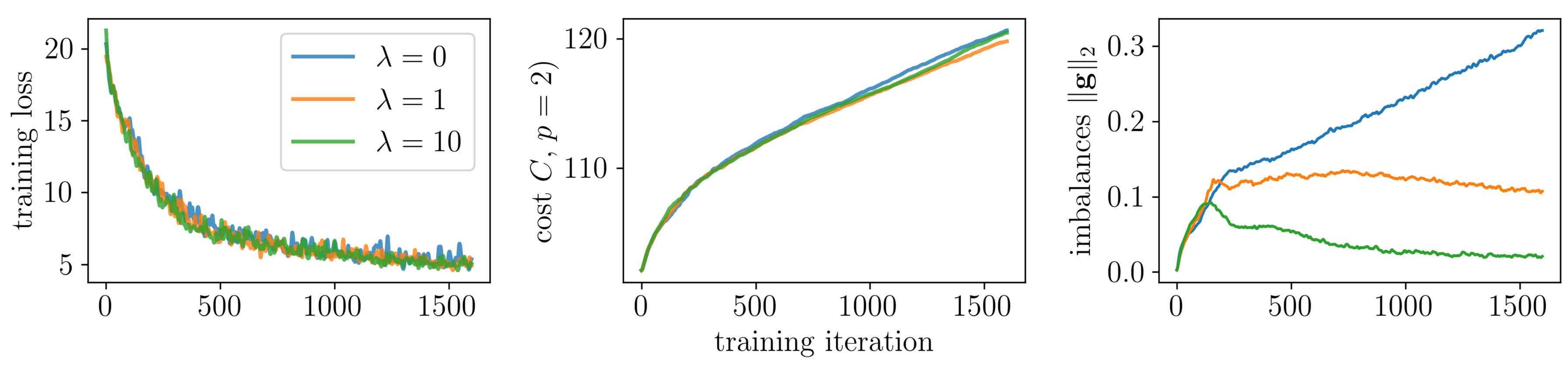
How much does this rule improve the cost, as we vary the rank of the network or the number of neurons?



What is the effect of this rule during learning?

Small network (N = 102) trained to perform context-dependent decision making [5].

Dynamics of learning = gradient descent + λ*(synaptic balancing)



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